MATHEMATICS II

029

20/11/2018 **8.30 AM-11.30 AM**

Rwanda Education Board

ADVANCED LEVEL NATIONAL EXAMINATIONS, 2018

SUBJECT: MATHEMATICS II

COMBINATIONS:

- MATHEMATICS-CHEMISTRY-BIOLOGY (MCB)
- MATHEMATICS-COMPUTER SCIENCE-ECONOMICS (MCE)
- MATHEMATICS-ECONOMICS-GEOGRAPHY (MEG)
- MATHEMATICS-PHYSICS-COMPUTER SCIENCE (MPC)
- MATHEMATICS-PHYSICS-GEOGRAPHY (MPG)
- PHYSICS-CHEMISTRY-MATHEMATICS (PCM)

DURATION: 3 HOURS

INSTRUCTIONS:

1) Write your names and index number on the answer booklet as written on your registration form and **DO NOT** write your names and index number on additional answer sheets of paper if provided.
2) Do not open this question paper until you are told to do so.
3) This paper consists of **two** sections: **A** and **B**.
    - **Section A**: Attempt **ALL** questions. **(55marks)**
    - **Section B**: Attempt **ONLY THREE** questions. **(45marks)**
4) **Geometrical instruments and silent non-programmable calculators may be used.**
5) Use only a **blue** or **black** pen.

SECTION A : ATTEMPT ALL QUESTIONS. (55 Marks)

1) Solve the inequality $|x-4| > |x+1|$. **(3marks)**

2) The polynomial $x^4 - 9x^2 - 6x - 1$ is denoted by $f(x)$.
   (a) Find the value of the constant a for which $f(x) = (x^2 + ax + 1)(x^2 - ax - 1)$. **(3marks)**
   (b) Hence solve the equation $f(x) = 0$, giving your answers in an exact form. **(2marks)**

3) A right triangle ABC, is rectangle in A such that $AB = 22\ cm$ and $AC = 50\ cm$.
   (a) Find the measure of the angle $A\hat{C}B$. **(2marks)**
   (b) Determine the real number n such that $BC = 2\sqrt{n}$. **(3marks)**

4) The parametric equations of a curve are $x = 2\vartheta - \sin 2\vartheta$; $y = 2 - \cos 2\vartheta$.
   (a) Show that $\frac{dy}{dx} = \cot\vartheta$. **(2marks)**
   (b) Find the equation of the tangent to the curve at the point where $\vartheta = \frac{\pi}{4}$. **(2marks)**
   (c) For the part of the curve where $0 < \vartheta < 2\pi$, find the coordinates of the points where the tangent is parallel to the x-axis. **(2marks)**

5) The equation of a curve is $y = \ln x + \frac{2}{x}$, where $x > 0$.
   Find the coordinates of the stationary point of the curve and determine whether it is a maximum or a minimum point. **(3marks)**

6) Events A and B are such that $P(A) = 0.3$, $P(B) = 0.8$ and $P(A \cap B) = 0.4$
   State, giving a reason in each case whether events A and $B$are:
   (a) Independent. **(1mark)**
   (b) Mutually exclusive. **(1mark)**

7) An arithmetic progression has the first term of 12 and the fifth term of 18. Find the sum of the first 25 terms. **(3marks)**

8) Find the value of the coefficient of $\frac{1}{x}$ in the expansion of $\left(2x - \frac{1}{x}\right)^5$. **(2marks)**

9) (a) Solve the equation $z^2 - 2iz - 5 = 0$, giving your answers in the form $x + yi$ where x and y are real. **(2marks)**

   (b) Find the polar form of each root. **(2marks)**

10) A curve is such that $\frac{dy}{dx} = 2x^2 - 5$.
    Given that the point (3,8) lies on the curve, find the equation of the curve. **(3marks)**

11) (a) Show that $\sin(x-60^{\circ})-\cos(30^{\circ}-x)=1$, can be written in the form $\cos x=k$, where k is a constant. **(3marks)**

(b) Hence solve the equation, for $0^{\circ}<x<180^{\circ}$. **(1mark)**

12) Find the angle between planes $\pi \equiv x+y+z=1$ and $\beta \equiv x-2y+3z=1$. **(3marks)**

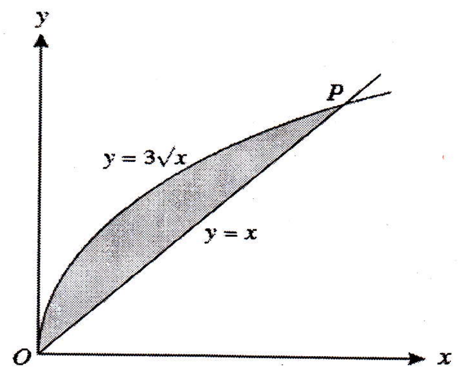
13) The line $x+2y=9$ intersects the curve $xy+18=0$ at the points A and B. Find the coordinates of A and B. **(5marks)**

14) Find the first derivative of $f(x)=(\sin x)^{\log x}$. **(3marks)**

15) It is given that $\int_0^a\left(\frac{1}{2}e^{3x}+x^2\right)dx=10$; where a is a positive constant. Show that $a=\frac{1}{3}\ln(61-2a^3)$. **(4marks)**

SECTION B: ATTEMPT ONLY THREE QUESTIONS. (45 Marks)

16) (a) The diagram below shows the curve $y=3\sqrt{x}$ and the line $y=x$ intersecting at O and P



Find:
(i) The coordinates of P. **(4marks)**
(ii) The area of the shaded region. **(4marks)**

(b) Determine the parametric and Cartesian equations of the line l which passes through point $(-3,5,7)$ and parallel to the line l' of the equations
$\begin{cases} x+y=1 \\ 4x-z=0 \end{cases}$ **(7marks)**

17) Consider the matrix $A=\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 5 & 7 \end{pmatrix}$

(a) Find the inverse A^{-1}. **(8marks)**

(b) Use the inverse A^{-1} and solve $\begin{cases} x+2y+3z=7 \\ 2x+3y+4z=10 \\ x+5y+7z=15 \end{cases}$ **(7marks)**

18) (a) A committee of 5 people is to be chosen from 6 men and 4women. In how many ways can this be done:

(i) If there must be 3 men and 2 women on the committee? **(3marks)**

(ii) If there must be more men than women on the committee? **(3marks)**

(iii) If there must be 3 men and 2 women and one particular woman refuses to be on the committee with one particular man? **(4marks)**

(b) By expanding $\sin(2x+x)$ and using double-angle formulae,

(i) Show that $\sin 3x = 3\sin x - 4\sin^3 x$ **(3marks)**

(ii) Hence show that $\int_0^{\frac{\pi}{3}} \sin^3 x dx = \frac{5}{24}$ **(2marks)**

19) In a certain chemical process, a substance is being formed, and t minutes after the start of the process there are m grams of the substance present. In the process, the rate of increase of m is proportional to $(50-m)^2$. When $t=0$, $m=0$ and $\frac{dm}{dt}=5$

(a) Show that m satisfies the differential equation $\frac{dm}{dt}=0.002(50-m)^2$ **(3marks)**

(b) Solve the differential equation and show that the solution can be expressed in the form $m = 50 - \frac{500}{t+10}$ **(4marks)**

(c) Calculate the mass of the substance when $t=0$ and find the time taken for the mass to increase from 0 to 45grams. **(4marks)**

(d) State what happens to the mass of the substance as t becomes very large. **(4marks)**

20) (a) Let $f(x) = \frac{4x}{(3x+1)(x+1)^2}$

(i) Express $f(x)$ in partial fractions. **(3marks)**

(ii) Hence show that $\int_0^1 f(x)dx = 1 - \ln 2$. **(3marks)**

(b) The equation of a curve is $y = 8x - x^2$

(i) Express $8x - x^2$ in the form $a-(x+b)^2$ stating the numerical values of a and b. **(2marks)**

(ii) Hence, or otherwise, find the coordinates of the stationary point of the curve. **(2marks)**

(iii) Find the set of values of x for which $y \geq -20$. **(5marks)**