

Mathematics VI

012

31 Oct 2007 8.30 am - 11.30 am



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ORDINARY LEVEL NATIONAL EXAMINATION 2007

SUBJECT : MATHEMATICS VI

TIME : 3 HOURS

INSTRUCTIONS:

- This paper consists of **TWO** Sections **A** and **B**.
- Attempt **ALL** questions in Section **A** and any **THREE** questions in Section **B**.
- Show **ALL** working clearly.
- Calculators and mathematical instruments may be used except when otherwise stated.
- Diagrams are not drawn to scale.

16.

a) Modal class = 45 - 49

Its limits = 44.5 - 49.5 or 45 - 49

b) the mean = 47

c)

Class	Mid-interval of class x	Difference from the working mean d	Frequency	fd
40 - 49	42	-5	15	-75
45 - 49	47	0	18	0
50 - 54	52	5	13	65
55 - 59	57	10	4	40

a) $\sum fd = -75 + 0 + 65 + 40 = 30$

b) Mean mass = $47 + \frac{30}{50} = 47.5\text{kg}$

17. a)

$$\overline{ABC} = \overline{BDC}$$

$$\overline{ACB} = \overline{BCD} \text{ (common angle)}$$

Triangle ABC and BDC share a common angle C, hence $\overline{ACD} = \overline{BCD}$, So $\overline{BAC} = \overline{CBD}$

b) i) $\frac{1}{2} \times 12 \times \overline{BC} = 54\text{cm}^2$

$$BC = \frac{54 \times 2}{12} = 9$$

$$AC = \sqrt{12^2 + 9^2} = \sqrt{144 + 81} = \sqrt{225} = 15\text{cm}$$

$$\frac{DC}{BC} = \frac{BC}{AC} = \frac{\overline{DC}}{9} = \frac{9}{15}$$

$$DC = \frac{81}{15} = 5.4\text{cm}$$

ii) $\frac{BD}{12} = \frac{9}{15}$

$$BD = \frac{9 \times 12}{15} = 7.2\text{cm}$$

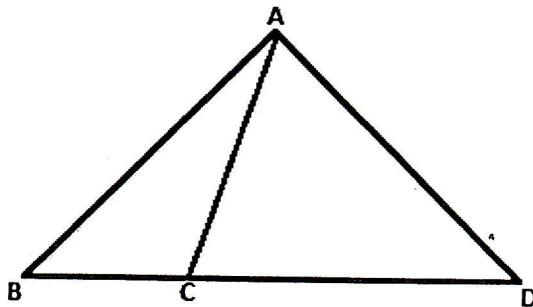
iii) Area of triangle ABD

$$= \frac{1}{2} \times 7.2 \times (15 - 5.4)$$

$$= \frac{7.2 \times 9.6}{15} = 34.56\text{cm}^2$$

SECTION A: Answer ALL questions. (55 marks)

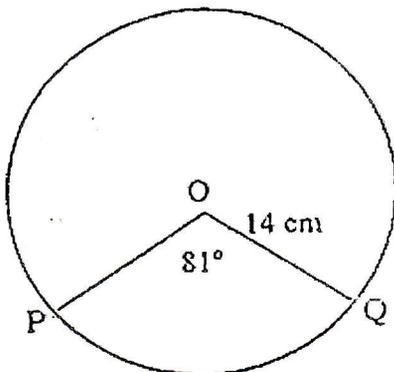
1. Calculate without using a calculator: $\frac{3.45^2 - 0.55^2}{4}$. (3 marks)
2. Find the equation of the line through the points (0, 2) and (2, 10). (3 marks)
3. Solve the quadratic equation: $2x^2 + 8x = -6$. (3 marks)
4. In a class of 40 pupils, 30 play football and 23 play basketball. 20 pupils play both games, how many pupils play neither? (3 marks)
5. If $f(x) = mx + n$, where m and n are integers and $f(0) = 2$, $f(-1) = -1$. Find m and n . (3 marks)
6. In the triangle ABD, angle ABD = x° , angle BAC = 40° , angle CAD = $2x^\circ$ and $\overline{AB} = \overline{AD}$. Find the size of angle ACD. (3 marks)



7. Solve the simultaneous equations:

$$2a + 3b = 16$$

$$4a + 5b = 28$$
 (4 marks)
8. 3 pupils share 36 sweets in the ratio $m : 3m : 5m$. How many sweets does each pupil get? (4 marks)
9. Solve the following inequality and illustrate the solution set on a number line:
 $0.25y - (y + 1) \geq 3$. (4 marks)
10. In the figure below, O is the centre of the circle. Calculate:
 (a) the length of the major arc PQ. (2 marks)
 (b) the area of the minor sector OPQ. ($\pi = \frac{22}{7}$). (2 marks)



11. 2,000,000 Rwf is invested at 12% per year compound interest. Find:

(a) the amount of investment after 3 year. (2 marks)

(b) the interest after 4 months. (2 marks)

12. Under enlargement the volume of a cube is 216cm^3 . If the volume of the object cube is 27cm^3 , find: a) Enlargement linear scale factor. (2 marks)

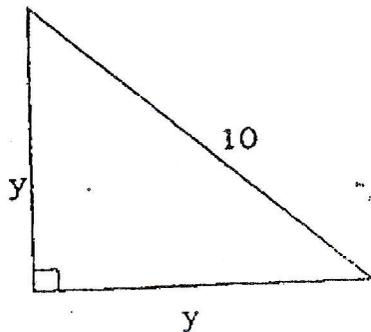
b) The total surface area of the enlarged cube. (2 marks)

13. Given vectors $\vec{a} = \begin{pmatrix} 7 \\ -3 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ and $\vec{c} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$. Find:

(a) the column vector for $\vec{a} + \vec{b} + \vec{c}$. (2 marks)

(b) the length of $\vec{a} + \vec{b} + \vec{c}$. (2 marks)

14. The figure below is a right angled triangle. Show that $y = 5\sqrt{2}$. (3 marks)



SECTION B: Answer THREE questions only. (45 marks)

16. a) Given the equation $2y + x = 6$

(i) Complete the table below (4 marks)

X	-2		2	4
Y		3		

(ii) Plot these points on the graph paper provided in your answer booklet and join them with a line. (6 marks)

(iii) Use letter A to indicate where you read the value of x when y = 0. (1 mark)

(iv) Write the value of x when y = 0. (1 mark)

(b) On the same graph as that of 16 (a) (ii) draw the graph of $y = 2$. Find the coordinates of intersection of $y = 2$ and $2y + x = 6$. (3 marks)

17. (a) Simplify: $\frac{2x+1}{3} - \frac{x+3}{2} + \frac{x+1}{6}$. (5 marks)

(b) Simplify completely: $\frac{3x^3-4x^2-x+2}{3x^2-6x+3}$. (10 marks)

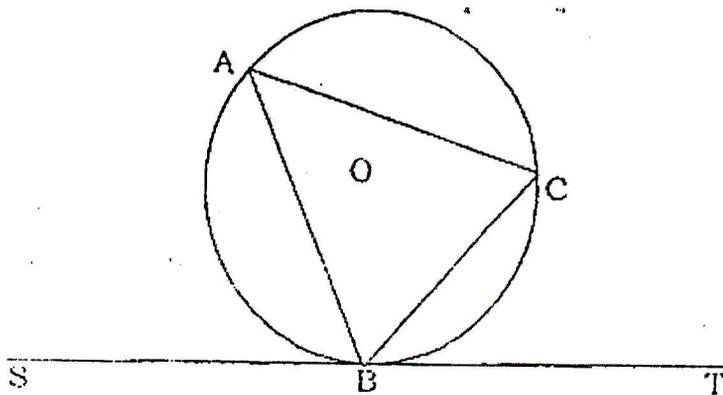
18. (a) Solve: $2x^4 + 4x^3 - 10x^2 - 12x = 0$.

(9 marks)

(b) One of the roots of $ax^2 + 5x + 2 = 0$ is -2, find the other root.

(6 marks)

19. In the figure below, O is the centre of the circle. The straight line SBT is a tangent to the circle.



(a) Show that angle CBT is equal to triangle BAC. Give reasons to support your answer. (10 marks)

(b) If angle ACB = 37° and angle BAC = 64° , find: i) angle ABS.

(2 marks)

ii) angle ABC.

(2 marks)

iii) angle CBT.

(1 mark)

20. The table below shows the marks scored by 25 pupils in a mathematics test marked out of 50.

27	35	40	42	28	40	28	42	27
28	28	35	40	42	35	40	28	35
27	40	28	35	40	28	42		

(a) Copy the table below and complete it using the above data.

(11 marks)

Mark (x)	Frequency (f)	Fx	Commutative frequency
	$\Sigma f =$	$\Sigma fx =$	

(b) Find the mode.

(5 marks)

(c) Find the median.

(5 marks)

(d) Find the mean.

(5 marks)

END

ANSWERS FOR NATIONAL EXAMINATION 2007.
MATHEMATICS VI
SECTION A

<p>1. $\frac{3.45^2 - 0.55^2}{4}$</p> $= \frac{(3.45+0.55)(3.45-0.55)}{4}$ $= \frac{4 \times 2.9}{4} = 2.9$	<p>2.</p> <p>The slope = $\frac{10-2}{2-0} = \frac{8}{2} = 4$</p> <p>The equation of the line is;</p> $y - 2 = 4(x-0)$ $y - 2 = 4x$ $y = 4x + 2$	<p>3.</p> $2x^2 + 8x = -6$ $= 2x^2 + 8x + 6 = 0$ $= 2x^2 + 6x + 2x + 6 = 0$ $= 2x(x+3) + 2(x+3) = 0$ $= (x+3)(2x+2) = 0$ $x = -3 \text{ or } x = -1$
<p>4.</p> <p>$n(F) = 30, n(B) = 23, n(F \cap B) = 20, n(C) = 40$ $C = 40$</p> <div style="display: flex; align-items: flex-start;"> <div style="border: 1px solid black; padding: 5px; margin-right: 10px;"> </div> <div style="margin-top: 10px;"> $x + 20 + 10 + 3 = 40$ $33 + x = 40$ $x = 7$ <p>7 pupils play neither.</p> </div> </div>		<p>5.</p> $f(0) = n = 2$ $f(-1) = -m + n = -1$ <p>Thus $-m + 2 = -1$</p> <p>$M = 3$ and $n = 2$</p>
<p>6.</p> <p>$x + x + 40 + 2x = 180^\circ$ because ABD is an isosceles triangle ($AB = AD = x$)</p> $4x = 140$ $x = 35$ <p>In triangle ACD; angle ACD</p> $= 180 - (2x + x)$ $= 180 - (70 + 35)$ $= 75^\circ$	<p>7.</p> $\begin{cases} 2a + 3b = 16 & \text{(i)} \\ 4a + 5b = 28 & \text{(ii)} \end{cases}$ $4a + 6b = 32$ $4a - 5b = 28,$ $b = 4$ <p>using equation (i),</p> $2a + 3 \times 4 = 16$ $a = 2$	<p>8. $m + 3m + 5m = 9m$</p> $= 9m = 36$ $m = 4 \text{ sweets}$ <p>1st pupil = $m = 4$ sweets</p> <p>2nd pupil = $3m = 4 \times 3 = 12$ sweets</p> <p>3rd pupil = $5m = 4 \times 5 = 20$ sweets</p>

		11. a) 1 st year = $\frac{2,000,000 \times 12 \times 1}{100} = 240,000$
9. $0.25y - (y + 1) \geq 3$ $\frac{y}{4} - y - 1 \geq 3$ $y = -4y - 4 \geq 12$ $-3y \geq 12 + 4$ $-3y \geq 16$ $y \leq \frac{16}{3}$	10. a) length of the major arc PQ $= 2 \times 14 \times \frac{22}{7} \times \frac{279}{360}$ $= 68.2\text{cm}$ b) the area of small sector $OPQ = 14 \times 14 \times \frac{81}{360} \times \frac{22}{7}$ $= 138.6\text{cm}^2.$	New principal = $2,000,000 + 240,000$ $= 2,240,000$ 2 nd year = $\frac{2,240,000 \times 12 \times 1}{100} = 268,800$ New principal = $2,240,000 + 268,800$ $= 2,508,800$ 3 rd year = $\frac{2,508,800 \times 12 \times 1}{100} = 301,056$ New amount after 3 years $= 2,508,800 + 301,056 = 2,809,856$ b) Interest after 4 months $\frac{2,000,000 \times 12 \times 4}{100 \times 12} = 80,000\text{Rwf}$
12. a) the homothetic ratio $= \frac{\sqrt[3]{216^3}}{273^3} = 2$ b) the enlarged edge of the cube = $\sqrt[3]{216^3} = 6\text{cm}$ TSA = $6 \times 6 \times 6 = 216\text{cm}^2$	13. a) $a + b + c$ $= \left(\frac{7}{-3}\right) + \left(\frac{-2}{3}\right) + \left(\frac{-2}{4}\right) = \frac{3}{4}$ b) $\sqrt{a+b+c} = \sqrt{3^2 + 4^2}$ $= \sqrt{9 + 16} = \sqrt{25}$ $= 5 \text{ units}$	14. $y^2 + y^2 = 10^2$ $2y^2 = 100$ $y = \sqrt{50}$ $= 5\sqrt{2}$
15. a) under a reflection $y = 2$: $R(2, -1)$	b) For a rotation of -90° on $(0,0);$ $(2, 5) \Leftrightarrow (5, -2)$	c) $T(2, 5) \Leftrightarrow (2+(-3)), (5 + (-1))$ $= (-1, 4)$

SECTION B

16.

a) i)

X	-2	0	2	4
Y	4	3	2	1

ii) ,iii), iv) and b : teacher's guidance

17.

a) $\frac{2x+1}{3} - \frac{x+3}{2} + \frac{x+1}{6}$

$= \frac{2(x+1) - 3(x+3) + (x+1)}{6} = \frac{4x+2-3x-9+x+1}{6}$

$= \frac{2x-6}{6} = \frac{2(x-3)}{6} = \frac{x-3}{3}$

18. a) $2x^4 + 4x^3 - 10x^2 - 12x = 0.$

=

	1	2	-5	-6
2	0	2	8	6
	1	4	3	0

$2x(x-2)(x^2+4x+3) = 0$

$2x(x^2 - 2)(x + 3)(x + 1) = 0$

$2x(x-2)(x+3)(x+1) = 0$

$2x = 0$ or $x - 2 = 0$ or $x = -3$ or $x = -1$

i.e $x = 0$ or $x = 2$ or $x = -3$ or $x = -1$

$S = \{-3, -1, 0, 2\}$

17. b)

$\frac{3x^3 - 4x^2 - x + 2}{3x^2 - 6x + 3}$

$= \frac{(x-1)(x-1)(3x+2)}{3(x-1)(x-1)}$

$= \frac{3x+2}{3}$

b) $= a(-2)^2 + 5(-2) + 2 = 0$

$= 4a - 10 + 2 = 0$

$= 4a = 8$

$= a = 2$

Thus $2x^2 + 5x + 2 = 0$

$2x^2 + 4x + x + 2 = 0$

$2x(x + 2) + (x + 2) = 0$

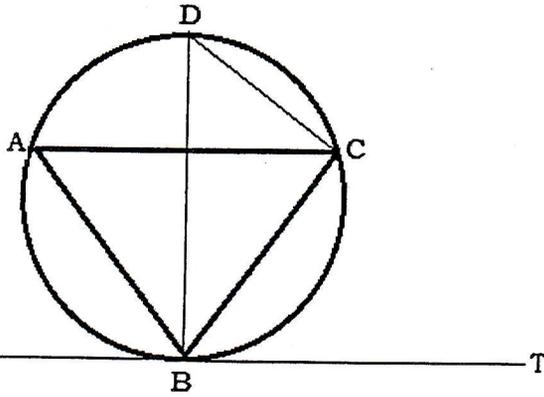
$(x + 2)(x + 2) = 0$

$x = -\frac{1}{2}$ or $x = -2,$

the other solution is $-\frac{1}{2}$

19.

a)



- Draw line BD through the centre O and attach D to C: $\angle BAC = \angle BDC$ because they intercept the same arc.
- $\angle CBT + \angle CBD = 90^\circ$ (diameter BD is tangent to SBT)
- $\angle BDC + \angle CBD = 90^\circ$, Thus $\angle CBT = \angle BDC$ but $\angle CBT = \angle BAC$, hence $\angle CBT = \angle BAC$.

b) i) $\angle ABS = \angle ACB = 37^\circ$

ii) $\angle ABS + \angle ABC + \angle CBT = 180^\circ$

thus $37^\circ + \angle ABC + 64 = 180^\circ$

$\angle ABC = 79^\circ$

iii) $\angle CBT = 64^\circ$

20.a)

Mark (x)	Frequency (f)	Fx	Commutative frequency
27	3	81	3
28	7	196	10
35	5	175	15
40	6	240	21
42	4	168	25
	$\sum f = 25$	$\sum fx = 860$	

b) Mode = 28

c) The median = 35

d) The mean = $\frac{860}{25} = 34.4$