Mathematics II	
029	
11/11/2015	8.30AM-11.30 AM

Rwanda Education Board

ADVANCED LEVEL NATIONAL EXAMINATIONS, 2015

SUBJECT: MATHEMATICS II

COMBINATIONS :

- **MATHEMATICS-CHEMISTRY-BIOLOGY (MCB)**
- **MATHS-COMPUTER SCIENCE-ECONOMICS (MCE)**
- **MATHEMATICS-ECONOMICS-GEOGRAPHY (MEG)**
- **MATHS-PHYSICS-COMPUTER SCIENCE (MPC)**
- **MATHEMATICS-PHYSICS-GEOGRAPHY (MPG)**
- **PHYSICS-CHEMISTRY-MATHEMATICS (PCM)**
- **PHYSICS-ECONOMICS-MATHEMATICS (PEM)**

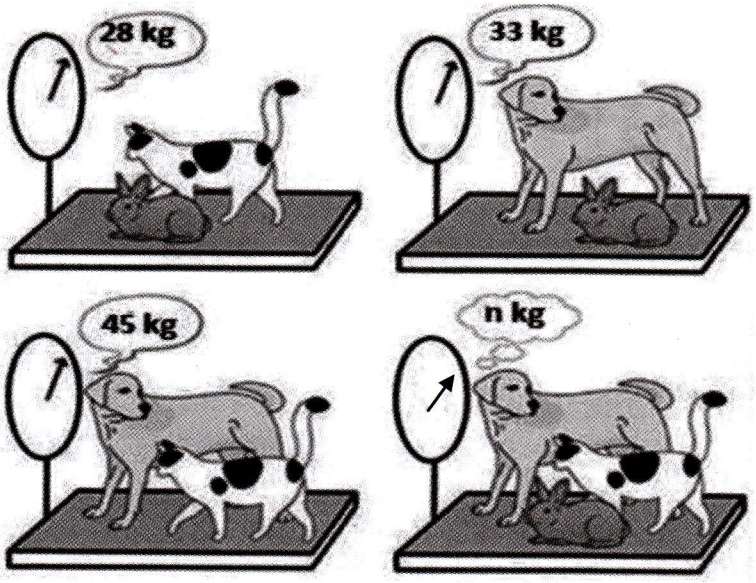
DURATION: 3 HOURS

INSTRUCTIONS :

1) Write your names and index number on the answer booklet as written on your registration form, and **DO NOT** write your names and index number on additional answer sheets of paper if provided.
2) Do not open this question paper until you are told to do so.
3) This paper consists of **two** sections: **A** and **B**.
   - **Section A**: Attempt **all** questions. **(55marks)**
   - **Section B**: Attempt **only three** questions. **(45marks)**
4) ***Geometrical instruments and silent non-programmable calculators may be used.***

SECTION A : ATTEMPT ALL QUESTIONS. **(55 MARKS)**

1)



Find the value of n. **(4marks)**

2) Differentiate $y = \left[2x^2 + \sqrt[3]{(3x-1)^4}\right]^n$, n is any constant. **(2marks)**

3) The members of a consulting firm rent cars from three rental agencies: 60 % on the cars from first agency, 30 % on the cars from second agency and 10 % on the cars from the third agency.

(a) If 9 % on the cars from the first agency, 20 % on the cars from the second agency and 6 % on the cars from the third agency need repairs, what is the probability that a rental delivered to the firm need repairs? **(2marks)**

(b) If a rental car delivered to the firm requires repairs, what is the probability that it is a car from the third agency? **(2marks)**

4) If the line through the points (4, 1, 2) and (5, n, 0) is parallel to the line through the points (2, 1, 1) and (3, 3, -1), find the value of n. **(3marks)**

5) Solve in IR: $4e^{3x} - 3e^{2x} - e^x = 0$ **(5marks)**

6) Let $A = 4sin^2\, x + 2cos^2\, x - 3$ with $x \in (-\pi, \pi)$

(a) Express A in terms of $\cos 2x$. **(4marks)**

(b) Determine the values of x for $A = \frac{1}{2}$ **(2marks)**

7) The values 4, 6, 12, 4, 10, 12, 3, x, y have a mean of 7 and 4 for mode.

(a) Find the value of x and y. **(2marks)**

(b) Find the median for the set of nine numbers given. **(2marks)**

8) For which value of k is the vector $\vec{u}\,(1, -2, k)$ of $\mathbb{R}^3$ a linear combination of vectors $\vec{v}$(3, 0, -2) and $\vec{w}$ (2, -1, -5)? **(3marks)**

9) Calculate $\int sinx \ln(1 + sinx)\, dx$ **(4marks)**

10) Calculate the first derivative of the function $f(x) = cosx - 3\sqrt[3]{x^2} - xe^x$ **(2marks)**

11) A mixed hockey team containing 5 men and 6 women is to be chosen from 7 men and 9 women. In how many ways can this be done? **(2marks)**

12) Consider the square matrix

$$M = \begin{pmatrix} 3 & 1 & -3 \\ 1 & 2a & 1 \\ 0 & 2 & a \end{pmatrix}$$

Find two values of a if M is singular matrix. **(4marks)**

13) The sequence V_n is defined as follows : $ln(7^n V_n) = 2n$

(a) Find V_0, V_1 and V_2. **(1mark)**

(b) Show that V_n is geometric sequence and find the common ration. **(1mark)**

14) Calculate :

(a) $\lim_{x \to -4} \frac{ln(x+5)}{x+4}$ **(2marks)**

(b) $\int_{-2}^{-1} \frac{1-x^2}{x^2} dx$ **(3marks)**

15) Suppose that $1 - 2i$ is a zero of the fourth – degree polynomial $f(x) = x^4 - 3x^3 + x^2 + 7x - 30$. Find all zeros of $f(x)$. **(3marks)**

SECTION B: ATTEMPT THREE QUESTIONS ONLY. (45marks)

16) (a) Given that $y = \sqrt{\frac{(x+1)(x+2)}{(x^2+1)(x^2+2)}}$, find y'. **(5marks)**

(b) A (1, -3) and B (4, 3) are two points on the curve y= $x - \frac{4}{x}$.

Find the coordinates of the arc AB of the curve at which the tangent to the curve is parallel to the line through A and B. **(5marks)**

(c) Using Taylor's series for $\sin x$ up to the 7th degree, evaluate $\int_0^1 \frac{\sin x}{x} dx$ and give your answer to two decimal places. **(5marks)**

17) (a) Express the complex number $3e^{\pi i}$ in the standard form $a + bi$. **(2marks)**

(b) Find all (real or complex) numbers x such that $x^3 = -8$ **(4marks)**

(c) Write an equation of the plane passing through P (1, 0, -1) with normal vector $\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$. **(4marks)**

(d) Discuss the domain of the function $f(x) = \sqrt{x^2 - 4nx + 5n - 1}$ in real variable x. **(5marks)**

18) a) The population (p) of enzymes in a culture solution changes according to the equation $\frac{dp}{dt} = \frac{3000}{1+0.25t}$, where t is the time in hours.

The initial population when $t = 0$ second is 1000.

i) Find the expression for the population (p) in terms of t. **(5marks)**

ii) Find the number of enzymes after t= 3 hours. **(5marks)**

b) Suppose that the profit (p) obtained in selling x units of certain item each week is given by $p = 50\sqrt{x} - 0.5x - 500$ where $0 \leq x \leq 8000$. Find the rate of change of p with respect to x when $x = 1600$. **(5marks)**

19) (a) In examination, Mahoro has to select 3 questions from each section. There are 5, 6 and 7 question in section A, section B and section C, respectively. What is the number of possible combinations in which she can choose the questions? **(5marks)**

(b) If the focus of a standard ellipse is at (1, 0) and corresponding directrix has the equation $x = 4$, find its equation. **(5marks)**

(c) Find the equation of the hyperbola in standard form whose eccentricity is $\sqrt{2}$ and the distance between whose foci is 16. **(5marks)**

20) Consider the function f defined on $\mathbb{R}$ by $f(x) = (x + \sqrt{x^2 + 1})^2$. Calculate the expression $(1 + x^2)f''(x) + xf'(x) - 4f(x)$. **(15marks)**