

**Mathematics II**

**029**

**30 Oct. 2013 8.30-11.30 am**

**REPUBLIC OF RWANDA**



**RWANDA EDUCATION BOARD**



**ADVANCED LEVEL NATIONAL EXAMINATIONS 2013**

**SUBJECT: MATHEMATICS II**

- COMBINATIONS: - MATHEMATICS-CHEMISTRY-BIOLOGY (MCB)**  
**- MATHS-COMPUTER SCIENCE-ECONOMICS (MCE)**  
**- MATHEMATICS-ECONOMICS-GEOGRAPHY (MEG)**  
**- MATHS-PHYSICS-COMPUTER SCIENCE (MPC)**  
**- MATHEMATICS-PHYSICS-GEOGRAPHY (MPG)**  
**- PHYSICS-CHEMISTRY-MATHEMATICS (PCM)**  
**- PHYSICS-ECONOMICS-MATHEMATICS (PEM)**

**DURATION: 3 HOURS**

**INSTRUCTIONS:**

1. Don't open this question paper until you are told to do so.
2. This paper consists of **two** sections: **A** and **B**.
  - **Section A:** Attempt **all** questions. **(55 marks)**
  - **Section B:** Attempt **only three** questions. **(45 marks)**
3. **Geometrical instruments and silent non-programmable calculators may be used.**

**SECTION A : ATTEMPT ALL QUESTIONS. (55 marks)**

- ✓1. Let consider the graph of the function  $f(x) = -2x^2 + 12x + 5$
- a) Specify the type of graph and explain why. (1 mark)
- b) Explain the concavity. (1 mark)
- c) Find the vertex. (2 marks)
2. Two cars start out at the same point. One car start out driving north at 25 km/h. Two hours later the second car starts driving east at 20 km/h. How long after the first car starts travelling does it take for the two cars to be 300 km apart? (4 marks)
3. Find the probability of getting 5 heads and 7 tails in 12 flips of a coin. (2 marks)
- ✓4. Solve for  $x$  :
- a)  $4^{5-9x} = \frac{1}{8^{x-2}}$  (3 marks)
- b)  $\log_2(x^2 - 6x) = 3 + \log_2(1 - x)$  (3 marks)
- ✓5. Evaluate the following limit:  $\lim_{x \rightarrow -\infty} \frac{\sqrt{3x^2+6}}{5-2x}$  (3 marks)
6. Find the derivative of the following function using the definition of the derivative:
- $$g(t) = \frac{t}{t+1}$$
- (3 marks)
7. A sphere was measured and its radius was found to be 45 cm with a possible error of no more than 0.01 cm. What is the maximum possible error in the volume if we use this value of the radius? (3 marks)
- ✓8. Evaluate the following integral:  $\int \frac{2t^2+1}{(t^2+2t)^2} dt$  (3 marks)



9. Solve:  $\cos x + \sin x > \sqrt{2}$  (Hint: use complex number theory). (3 marks)

10. Using integral rules, compute the circumference of a circle centred at the origin. (6 marks)

11. Calculate the expected value, the variance and the standard deviation for the following probability density function:

$x_i$	-5	-4	1	2
$f(x_i)$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{2}$	$\frac{1}{8}$

(3 marks)

12. Solve for z:  $|3 - 2z| \leq 5$  (3 marks)

13. Find the slope and the y-intercept of the regression line  $y = ax + b$  that fits the following data: (4 marks)

Marks(x)	5	5	7	7	9	11	13	15	14	13	16	17
Marks(y)	4	8	10	7	10	10	12	13	15	16	17	17

14. Solve:  $2(\ln x)^3 + (\ln x)^2 - 5 \ln x + 2 = 0$  (4 marks)

15. Determine the shape of the graph with the following equation: (4 marks)

$$x^2 + y^2 - 3x + 10y - 1 = 0$$

Find its parameters.

**SECTION B: Attempt ONLY THREE questions (45 marks)**

16. a) Solve the following system of equations using the Gauss-Jordan elimination method.

$$\begin{cases} 3x + y - 2z = 2 \\ x - 2y + z = 3 \\ 2x - y - 3y = 3 \end{cases}$$

(7 marks)

(8 marks)

b) Solve:  $y'' - y' - 2y = \sin 2x$

17. a) Solve for t:  $4\sin^2\left(\frac{t}{3}\right) - 3\sin\left(\frac{t}{3}\right) = 1$  (7 marks)

$n = 25$   
 $\frac{x}{y} = \frac{z}{t}$

- b) Determine if the following vectors are in the same plane: **(8 marks)**  
 $\vec{a}(1;4;-7)$  ;  $\vec{b}(2;-1;4)$  ;  $\vec{c}(0;-9;18)$ .
18. Calculate the area of the surface bounded by the curve :
- a)  $x = 6(\theta - \sin \theta)$  and  $y = 6(1 - \cos \theta)$ , where  $0 \leq \theta \leq 2\pi$ . **(8 marks)**
- b)  $f(x) = \ln \frac{1+x}{1-x}$ , where  $\frac{\pi}{2} \leq \theta \leq 2\pi$  **(7 marks)**
19. a) Determine the standard form of the following flat shape : **(5 marks)**  
 $5x^2 + y^2 - 10x + 4y + 4 = 0$
- b) Which shape is it? **(1 mark)**
- c) Find the coordinates of the most points, the focal points, and the equation of the focal line. **(9 marks)**
20. Compute:
- a)  $\int \sin x e^{\cos x} dx$  **(5 marks)**
- b)  $\int \sin x * \sin 3x dx$  (Hint: apply complex numbers). **(10 marks)**