Mathematics II
047
03 Nov. 2010 08.30 - 11.30 am

RWANDA NATIONAL EXAMINATIONS COUNCIL

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ADVANCED LEVEL NATIONAL EXAMINATIONS 2010

SUBJECT: MATHEMATICS II

COMBINATIONS:
- MATHS-CHEMISTRY-BIOLOGY: MCB(E)
- MATHS-ECONOMICS-GEOGRAPHY: MEG(E)
- MATHS-PHYSICS-GEOGRAPHY: MPG(E)
- PHYSICS-CHEMISTRY-MATHS: PCM(E)
- MATHS-PHYSICS-COMPUTER SCIENCE: MPC(E)
- MATHS-COMPUTER SCIENCE-ECONOMICS: MCE(E)

TIME : 3 HOURS

INSTRUCTIONS :

- This paper consists in **two** sections: **A** and **B**.

Section A : Attempt **all** questions. **(55 marks)**
Section B: Attempt any **three** questions. **(45 marks)**

Geometrical instruments and calculators may be used.

SECTION A: Attempt all questions. **(55 marks)**

1. In an arithmetical progression, the thirteenth term is 27 and the seventh is three times the second. Find the first term, the common difference and the sum of the first ten terms. **(5 marks)**

2. Find the equation of the normal to the curve $2x^2-6xy+y^2=9$ in the point (4,1). **(5 marks)**

3. Two machines A and B produce 60% and 40% respectively of the total output of a factory. Of the parts produced by machine A, 3% are defective and of the parts produced by machine B, 5% are defective. A part is selected at random from a day's production and found to be defective. What is the probability that it came from machine A? **(4 marks)**

4. The amount *A(t)*, in grams, of radioactive material in a sample after t years, is given by $A(t)=80(2^{-t/100})$.
   (a) Find the amount of material in the original sample. **(1 mark)**
   (b) Calculate the half-life of the material. [The half-life is the time taken for half of the original material to decay]. **(2 marks)**
   (c) Calculate the time taken for the material to decay to 1 gram. **(2 marks)**

5. Find the angle between the lines

$$\frac{x+2}{2}=\frac{y+1}{2}=-z \text{ and } \frac{x+2}{3}=\frac{y}{6}=\frac{z-1}{2}.$$

(4 marks)

6. Suppose that the profit P obtained in selling x units of a certain item each week is given by
   $P=50\sqrt{x}-0{,}5x-500$, $0\le x\le 8000$.
   Find the rate of change of P with respect to x when x=1600. **(2 marks)**

7. Find the intervals for which the following function is Continuous:

$$f(x)=\begin{cases} x\sin\frac{1}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}$$

(2 marks)

8. Given the function $f(x)=5-\frac{4}{x}$, find all c in the interval $[1,4]$ such that $f'(c)=\frac{f(4)-f(1)}{4-1}$.

(3 marks)

9. Find the volume of the solid formed by revolving the region bounded by the graphs of $y=x^2+1$, $y=0$, $x=0$ and $x=1$ about the y-axis.

(4 marks)

10. Prove that the function $f: IR \to IR : x \to f(x)=x^2$ is neither an injection nor a surjection.

(2 marks)

11. Evaluate the following limit: $\lim_{n\to\infty}\prod_{k=1}^{\infty}\left(1+\frac{k}{n^2}\right)$. **(4 marks)**

12. Evaluate the integral $\int_{-1}^{1}\frac{dx}{x^2-2x\cos\alpha+1}$ $\quad(0 \prec \alpha \prec \pi)$ **(4 marks)**

13. Show the polynomial $T_m(x)=\frac{1}{2^{m-1}}\cos(m\arccos x)$ $\quad(m=1,2,3,\ldots)$, satisfies the following differential equation:
$(1-x^2)T_m''(x)-xT_m'(x)+m^2T_m(x)=0$.

(3 marks)

14. (a) Express $\delta=\sin^2 t-2(1-\cos t)$ in terms of $\sin\frac{t}{2}$ $\quad(t\in IR)$. **(2.5 marks)**

(b) Solve the equation $2u(1-\cos t)-2u\sin t+1=0$, $u\in Z$ **(2.5 marks)**

15. Suppose that $a\equiv b(\text{mod } m)$ & $c\equiv d(\text{mod } m)$.
Prove that:

(i) $a+c\equiv b+d(\text{mod } m)$ **(1.5 marks)**

(ii) $ac\equiv bd(\text{mod } m)$ **(1.5 marks)**

SECTION B : Attempt any three questions. **(45 marks)**

16. A) At noon, two boats P and Q are at points whose position vectors are $4i+8j$ and $4i+3j$ respectively. Both boats are moving with constant velocity; the velocity of P is $4i+j$ and the velocity of Q is $2i+5j$, (all distances are in kilometres and the time is measured in hours). Find the position vectors of P, Q and $\overrightarrow{PQ}$ after t hours and hence express the distance PQ between the boats in terms of t. Find the least distance between the boats. **(7.5 marks)**

B) The random variable X has the following probability distribution:

x	P(X=x)
2	a
4	$2a^2-a$
6	a^2+a-1

Find:

(a) the value of a. **(3 marks)**

(b) E(X). **(1 mark)**

(c) V(X). **(2 marks)**

(d) SD(X). **(1.5 marks)**

17. (a) Find the shortest distance between the skew lines

$$L_1 : \begin{cases} x = 5 + 2\lambda \\ y = 3 - \lambda \\ z = 0 \end{cases} \text{ and } L_2 : \begin{cases} x = 2 \\ y = \mu \\ z = 9 - \mu \end{cases}$$ **(8 marks)**

(b) If $A = \begin{pmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 2 & -1 & -2 \end{pmatrix}$, evaluate A^3 and hence find A^{-1}. **(7 marks)**

18. Find the particular solution to the following differential equation, which satisfies the given initial values:
$y''-2y'=x+2e^{x}$; $y(0)=0$; $y'(0)=1$ **(15 marks)**

19. (a) Sketch the graph of the polar equation

$$r=\frac{-32}{3-5\sin\theta}$$ **(7.5 marks)**

(b) Determine the convergence or divergence of

$$\sum_{n=1}^{\infty}\frac{n^{3}}{3^{n}}.$$ **(7.5 marks)**

20. Let $A=\begin{pmatrix}5 & 2 & -2\\ 2 & 5 & -2\\ -2 & -2 & 5\end{pmatrix}$

(a) Verify that $\det(\lambda I_3-A)$, the characteristic polynomial of A, is given by $(\lambda-3)^{3}(\lambda-9)$. **(4 marks)**

(b) Find a non-singular matrix P such that $P^{-1}AP=diag(3,3,9)$. **(11 marks)**