Mathematics II

027

02 Nov. 2011 08.30am - 11.30am

REPUBLIC OF RWANDA

RWANDA EDUCATION BOARD (REB)
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ADVANCED LEVEL NATIONAL EXAMINATIONS 2011

SUBJECT: MATHEMATICS II

COMBINATIONS:

- **MATHS-CHEMISTRY-BIOLOGY: MCB**
- **MATHS-COMPUTER SCIENCE-ECONOMICS: MCE**
- **MATHS-ECONOMICS-GEOGRAPHY: MEG**
- **MATHS-PHYSICS-COMPUTER SCIENCE: MPC**
- **MATHS-PHYSICS-GEOGRAPHY: MPG**
- **PHYSICS-CHEMISTRY-MATHS: PCM**
- **PHYSICS-ECONOMICS-MATHS: PEM**

TIME: 3 HOURS

INSTRUCTIONS:

- This paper consists in **two** sections: **A** and **(B)**

Section A : Attempt **all** questions. **(55 marks)**
Section B: Attempt any **three** questions. **(45 marks)**

Geometrical instruments and silent non-programmable calculators may be used.

SECTION A: Attempt all questions. **(55 marks)**

1. What values (real numbers) of x satisfying the following condition:
   (a) $4(x+5)-6(2x+3)=3(x+14)-2(5-x)+9$ **(2.5 marks)**
   (b) $|6-3x| \succ 14$ where $|a|$ stands for absolute value of *a defined* as $|a|=\begin{cases} a \text{ if } a \geq 0 \\ -a \text{ if } a \leq 0 \end{cases}$ **(2 marks)**

2. If (u,v,w) is a basis of the real vector space $\Re^3$ determine whether or not $(u+v, u+2w, u-w)$ is also a basis of $\Re^3$. **(3 marks)**

3. Let $f:\Re \to \Re : f(x)=\dfrac{x^2-1}{|x-1|}$
   (a) Find $\lim_{x\to 1>} f(x)$ *and* $\lim_{x\to 1<} f(x)$ **(3.5 marks)**
   (b) Discuss the limit of *f(x)* as *x* approaches 1. **(0.5 mark)**
   (c) Sketch the graph of *f(x)*. **(1 mark)**

4. Express $f(x)=\dfrac{3x^3-x^2-13x-13}{x^2-x-6}$ in partial fractions.
   Then find antiderivative of *f(x)*. **(4.5marks)**

5. Find the number of ways that 6 teachers can be assigned to 4 sections of mathematics course if no teacher is assigned to more than one section. **(2.5 marks)**

6. In Euclidian space, find an equation for the plane consisting of all points that are equidistant from the points (-4, 2, 1) and (2, -4, 3). **(3 marks)**

7. Find any asymptotes of the function f if $f(x)=\dfrac{x-1}{x^2-1}$ **(3marks)**

8. Find a second degree polynomial P (x) such that P(2)=5, P'(2)=3 and P"(2)=2 where P' and P" are first and second derivatives of P respectively . **(4 marks)**

9. (a) Evaluate the derivative of $f(x)=\ln(x^3+7x^2)$ where **ln** stands for natural logarithm function; **(1.5 marks)**
   (b) and evaluate the integral $\int \dfrac{x^3}{x^4+7}dx$ **(1.5 marks)**

10. How many distinct permutations can be made from the letters of the word **"infinity"**? **(2marks)**

11. Find the domain and the derivative of the numerical function f, if $f(x)=\dfrac{x}{1-\ln(x-1)}$. **(4marks)**

12. The probability that a patient recovers from a delicate heart operation is 0.8. What is the probability that
    (a) Exactly 2 of the next 3 patients who have this operation survive? **(1.5 marks)**
    (b) All of the next 3 patients who have this operation survive? **(1.5 marks)**

13. Find the value of the complex number $Z=\left(-\dfrac{1}{2}+i\dfrac{\sqrt{3}}{2}\right)^{2010}$.
    Leave your answer in standard form $Z=a+b\,i$. **(4marks)**

14. Sketch the plane region bounded by $y=x$ *and* $y=x^2-6$.
    Then estimate the volume generated by this region when revolved about $x=-3$. **(5.5 marks)**

15. The numbers of incorrect answers on a true-false competency test for a random sample of 15 students were recorded as follows: 2, 1, 3, 0, 1, 3, 6, 0, 3, 3, 5, 2, 1, 4, and 2.
    Find the mean, the median, the mode and the sample standard deviation. **(4 marks)**

SECTION B: Attempt any three questions. **(45 marks)**

16. Let $f:\Re\to\Re: f(x)=|x-2|-1+\dfrac{1}{x^2}$
    (a) Determine the domain of *f(x)*. **(2 marks)**
    (b) Write f (x) without signs of absolute value. **(2 marks)**
    (c) Study the derivability of *f(x)* at *x=-2* . **(4 marks)**
    (d) Evaluate the limit of *f(x)-(-x+1)* when x approaches $-\infty$ and the limit of *f(x)-(x-3)* when x approaches $+\infty$.
    Is there any relationship between lines y=x-3 and y=-x+1, and the graph of the function *f*? **(3 marks)**
    (e) Evaluate $\int_1^3 f(x)dx$. **(4 marks)**

17. (a) Solve the equation ${}^{n-1}C_{n-5}=3\,{}^{n-3}C_{n-7}$ (or $\binom{n-1}{n-5}=3\binom{n-3}{n-7}$)
    in the set of positive integers. **(6.5marks)**

(b) Consider the quadratic polynomial $z^2 - 6z + c$ where c is real. For what values of c does this polynomial have real roots? **(2.5 marks)**

(c) Multiply out the expression $(z+7)(z^2-6z+25)$ and hence find all roots (real or complex) of the polynomial $z^3 + z^2 - 17z + 175$. **(6 marks)**

18. (a) In Euclidian space, find vector, parametric and symmetric equations for

(i) the line through origin and the point (1,2,3) **(4marks)**

(ii) The line through (0,2,-1) and parallel to the line with parametric equations $\begin{cases} x = 1 + 2t \\ y = 3t \\ z = 5 - 7t \end{cases}$ **(4 marks)**

(b) Find all cube roots of the complex number $W = -1 + i$. Leave your answer in polar form and trigonometric form. **(7 marks)**

19. (a) Compute the sixth degree Taylor polynomial generated by $f(x) = \ln x$ about $x=1$.

Using this result, evaluate $\lim_{x \to 1} \frac{\ln x}{x-1}$. **(6marks)**

(b) Find all values of x that satisfy

(i) the inequality $2\cos(x) + 1 \geq 0$ in the interval $[0, 2\pi]$; **(2.5 marks)**

(ii) the equality $\log_2 \frac{x^2 - 1}{x+1} = 1$ **(3.5 marks)**

(c) If 3 books are picked at random from a shelf containing 5 novels, 3 books of poems, and one dictionary, what is the probability that

(i) The dictionary is selected? **(1.5 marks)**

(ii) 2 novels and 1 book of poems are selected? **(1.5 marks)**

20. (a) Let * be a binary operation defined on the set Z of all integers by $x * y = x + y + 3$.

Determine whether the operation is commutative, and whether there is an identity element.

Can you find a symmetric (inverse) of any integer? **(8 marks)**

(b) Sketch and estimate the area of the region bounded by the curves $y = x$ *and* $y = x^2 - 2$. **(7 marks)**