ORDINARY LEVEL NATIONAL EXAMINATION 2011

SUBJECT: MATHEMATICS 1

DURATION: 3 HOURS

INSTRUCTIONS:

- This paper has TWO sections A and B.

**SECTION A:** Answer ALL questions. (55 marks)

**SECTION B:** Answer any THREE questions. (45 marks)

- Calculators and mathematical instruments may be used where necessary.

- Show all the working. **No marks will be given for answers which do not show all the steps.**
SECTION A: ATTEMPT ALL QUESTIONS (55 MARKS)

1. In a school there is enough food to feed 200 students for 15 days. For how long will the food last if 50 more students join the school? (3 marks)

2. Some money was invested at 15% per year simple interest for 3 years. If the interest is 7200frw, calculate the investment. (3 marks)

3. A \((1, 4), B (1, 0)\) and C \((3, -2)\) are three of the vertices of a quadrilateral ABCD. \(\overrightarrow{AB} = (4, 1)\) and X is the mid-point of \(\overrightarrow{AC}\). Find the coordinates of X and D. (4 marks)

4. 80 pupils had meals at a hotel. Of these 50 had a meal of rice \((R)\) and 45 had a meal of potatoes \((P)\).
   (a) Represent this information in a Venn diagram. (2 marks)
   (b) Find the number of people who had a meal of \(R\) and \(P\). (2 marks)

5. If the gradient of a line is 4, find the equation of this line if it passes through points \((5, 3)\) and \((x, y)\). (3 marks)

6. Think of a number, multiply it by 30 and then add 32. The result is equal to twice the square of the number. Find the number if it is greater than 0. (3 marks)

7. Given that \(f(x) = \frac{8}{1-x^2}\) and \(f(x) = -1\), find \(x\). For which values of \(x\) is \(f(x)\) not defined? (4 marks)

8. The diameters of two cylinders are in the ratio 3:4. The diameter of the smaller cylinder is 15cm.
   (a) Find the circumference of the larger cylinder. \(\pi = \frac{22}{7}\). (2 marks)
   (b) What is the ratio of the area of the curved surfaces of the cylinders? (2 marks)

9. Solve the following inequality and illustrate the solution on the number line: \(\frac{1}{3} x^2 - (x+1) \geq 3\). (4 marks)

10. In a triangle ABC, \(AB = 7.3\) cm, \(BC = 6.0\) cm and \(AC = 5.0\) cm. Using a ruler and a pair of compasses only, draw accurately triangle ABC. Find the height of the triangle by construction and calculate its area. (4 marks)

11. Solve the following simultaneous equations:
   \[
   \begin{align*}
   4y - 3x &= 2 \\
   2y + 1 &= 2x
   \end{align*}
   \] (4 marks)

12. Three lights flash at intervals of 4, 6 and 10 seconds respectively. If they are started together, how soon after will they next flash together again? (3 marks)
From your diagram, find the distance and bearing of K from the position of the helicopter. (4 marks)

(c) Given that the helicopter flies at a steady speed of 200 km/hr, Find how long the whole journey took. (2 marks)

18. Draw on the same axes the following graphs: (a) \( y = 2x + 6 \) and (b) \( y + x = 3 \)

From the graph determine the coordinates where the two lines intersect. (15 marks)

19. In the circle below \( FH \) is the diameter of the circle, \( EF = r, \EH = s, \FG = x \) and \( GH = y \).

(a) Show that \( y^2 = r^2 + s^2 - x^2 \). State your reasons clearly. (9 marks)

(b) Given that \( r = 8 \text{ cm} \) and \( s = 6 \text{ cm} \), find \( FH \). (3 marks)

(c) Find the area of triangle \( FGH \), if triangle \( FGH \) is an isosceles triangle. (3 marks)

20. Q (1, 3) R (4, 3) and S (4, -3) are vertices of triangle QRS.

(a) Plot on a graph paper points Q, R and S. Join these points to form triangle QRS. (5 marks)

(b) Triangle QRS is translated by \( T = \begin{pmatrix} -1 \\ -3 \end{pmatrix} \). Find the coordinates of \( Q', R', S' \) the images of Q, R and S under translation T. Plot these points on the same graph as (a) and join them to form triangle \( Q'R'S' \). (4 marks)

(c) Rotate \( Q'R' \) and \( S' \) about the origin through -90°. Find points \( Q'', R'' \) and \( S'' \). Plot these points on the same graph as (a) and join them. (3 marks)

(d) Find the two successive transformations that would map triangle \( Q''R''S'' \) on to triangle QRS. (2 marks)

END
### SECTION A

**1.**
- For 200 students, food lasts 15 days.
- For 1 student, food lasts 15 days \times 200
- For 250 students: 
  \[
  \frac{15 \text{ days} \times 200}{250} = 12 \text{ days}
  \]

**2.**
- For 200 students, food lasts 15 days.
- For 1 student, food lasts 15 days \times 200
- For 250 students: 
  \[
  \frac{15 \text{ days} \times 200}{250} = 12 \text{ days}
  \]

\[
I = \frac{P \times R \times T}{100}
\]

\[
7200 = \frac{P \times 15 \times 3}{100}
\]

\[
P = 16,000 \text{ frw}
\]

**3.**
A(1, 4), B(1, 0), C(3, -2)

\[
\overrightarrow{AD} = \left(\begin{array}{c}
4
\end{array}\right), \quad AD = d - a
\]

\[
= \left(\begin{array}{c}
4
\end{array}\right) - \left(\begin{array}{c}
1
\end{array}\right)
\]

\[
= \left(\begin{array}{c}
3
\end{array}\right)
\]

D = (5, 3)

Midpoint of \overline{AC} = x

\[
= \frac{x_1 + x_2}{2}, \quad \frac{y_1 + y_2}{2}
\]

\[
= \frac{3 + 1}{2}, \quad \frac{-2 + 4}{2}
\]

\[
= (2, 1)
\]

**4.**

| a) | b) 
|---|---
| E = 80 | Number of people who had a meal of R and P 
| n(R) = 50 | = 50 - x + x + 45 - x = 80 
| n(P) = 45 | \[x = 15\] 
| 50-x | 45-x |

**5.**

\[y = mx + C\]

\[m = 4\]

\[3 = 4(5) + C\]

\[3 = 20 + C\]

\[C = -17\]

\[y = 4x - 17\]

The equation if the line is \[y = 4x - 17\].

**6.**

\[30x + 32 = 2x^2\]

\[30x + 32 - 2x^2 = 0\]

\[-2x^2 + 30x + 32 = 0\]

\[-2x^2 + 32x - 2x + 32 = 0\]

\[-2x(x - 6) - 2(x - 16) = 0\]

\[(x - 16) (-2x - 2) = 0\]

Either \[x - 16 = 0\]

\[x = 16\]

Or \[-2x - 2 = 0\]

\[-2x = 2\]

\[x = -1\]

\[x = 16 \text{ or } -1\]

But the number is 16 since \[x > 0\]
\[
\frac{8}{1-x^2} = -1 = 8 \\
= -1(1-x^2) \\
8 = -1 + x^2 = 0 \\
= -1 + x^2 - 8 \\
0 = x^2 - 9 \\
= (x + 3)(x - 3) = 0 \\
x = -3 \text{ or } x = 3
\]

a) the diameter of the larger cylinder
\[= 15 \text{cm} \times \frac{22}{7} = 20 \text{cm}\]

Circumference = \(\pi D = \frac{22}{7} \times 20\)
\[C = \frac{440}{7} = 62.8 \text{cm}\]

b) ratio of the area of the curved surfaces of the cylinders
\[\frac{3^2}{4^2} = \frac{9}{16}\]

ratio = 9 : 16

\[
\frac{1}{3} x - (x+1) \geq 3 \\
\frac{1}{3} x - x - 1 \geq 3 \\
\frac{1 - 3x \geq 12}{3} = x \geq -6
\]

Area of triangle ABC
\[= \frac{1}{2} \times 7 \times 4.1 = 14.4 \text{cm}^2\]

11. \[\begin{array}{l}
4y - 3x = 2 \\
2y + 1 = 2x \\
4y - 3x = 2 \\
-4y - 4x = -2 \\
\end{array}\]
\[y = 3.5\]
### 12.

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<th>4</th>
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\[
\text{LCM} = 2^2 \times 3 \times 5 = 60
\]

The lights will together flash again after 60 seconds.

### 13.

a) \[
\frac{PE}{DQ} = \frac{2\text{cm} \cdot 6\text{cm}}{4\text{cm}} = 3\text{cm}
\]

b) \[
\frac{DP \cdot EF}{DE} = \frac{4\text{cm} \cdot 12}{6\text{cm}} = 8\text{cm}.
\]

### 14.

\[
KL^2 + KM^2 = LM^2
\]

\[
KL^2 = 9^2 - 6^2 = 45\text{cm}^2
\]

\[
KL = \sqrt{45} = 3\sqrt{5}\text{cm}
\]

\[
\frac{LP}{KL} = \frac{KL}{LM} \Rightarrow LP = \frac{KL}{LM} = \frac{45}{9} = 5\text{cm}
\]

### 15.

a) Mean mark = \[
\frac{\sum fx}{\sum f} = \frac{127}{25} = 5.08
\]

b) median mark = 5

c) mode mark = 7
SECTION B

16.
Let the width be \( x \)

\[
\begin{array}{c}
2x + 1 \\
x + 2
\end{array}
\]

The length of is \( (x + 2) \)cm

The height is \( (2x + 1) \)cm

The volume is \( x(x+2)(2x+1) \)

\( V = \text{base area} \times \text{height} \)

\[624 = 2x^3 + x^2 + 4x^2 + 2x\]

\[624 = (x + 2) \times x \times (2x + 1)\]

\[624 = (x^2 + 2x)(2x + 1)\]

\[624 = 2x^3 + x^2 + 4x^2 + 2x\]

\[624 = 2x^3 + 5x^2 + 2x\]

\[ x = 6 \]

\[ x - 6 = 0 \]

\[
\begin{array}{c}
\sqrt{2x^2 + 17x + 103} \\
x - 6 \quad 2x^3 + 5x^2 + 2x - 624 \\
- 2x^3 - 12x^2 \\
\quad 17x^2 + 2x \\
- 17x^2 - 102x \\
\quad 104x - 624 \\
\quad - 104x - 624 \\
\quad 0
\end{array}
\]

\[2x^3 + 5x^2 + 2x - 624 = (x - 6)(2x^2 + 17x + 104)\]

\[(x - 6)(2x^2 + 17x + 104) = 0\]

One of the possible values of \( x = 6 \)

\( L = 8 \text{cm}, W = 6 \text{cm}, H = 13 \text{cm}.\)

T.S.A = \( 2(L \times W) + 2(L \times H) + 2(W \times H) \)

\[2(8 \times 6) + 2(8 \times 13) + 2(6 \times 13)\]

\[(2 \times 48) + (2 \times 104) + (2 \times 78)\]

\[= 96 + 208 + 156\]

\[= 460 \text{cm}^2\]
17. 50km = 1cm
   200km = 4cm
   250km = 5cm
   300km = 6cm
   a) Distance = 2.2cm = 2.2 \times 50 = 110km
   Bearing = 90°
   c) s = 200km/h
   \[
s = \frac{D}{T}
\]
   \[
s = \frac{750}{200} = 3hrs\ 45\ min
\]

18. Teacher's guidance

19. 
   a) In \triangle EFH, FH is the diameter,
   Angle FEH = 90° (because the angle subtends
   the diameter at the circumference)
   \[
   FH^2 = FE^2 + EH^2
   = r^2 + s^2
   \]
   \[
   FGH = 90°
   FH^2 = FG^2 + GH^2
   = x^2 + y^2
   \]
   \[
   r^2 + s^2 = x^2 + y^2
   \]
   So \[r^2 + s^2 - x^2 = y^2\]
   \[
   y^2 = r^2 + s^2 - x^2
   \]

   b) \[
   FH^2 = r^2 + s^2
   \]
   \[
   s^2
   \]
   \[
   FH^2 = 8^2 + 6^2
   \]
   \[
   FH^2 = 100
   \]
   \[
   FH = 10cm
   \]

   c) FG = GH \iff x
   \[
   FH = x^2 + x^2
   \]
   \[
   100 = 2x^2
   \]
   \[
   x = \sqrt{50}
   \]
   \[
   Area = \frac{\sqrt{50} \times \sqrt{50}}{2} = \frac{50}{2}
   \]
   \[
   Area = 25cm^2
   \]

20. Teacher's guidance